



MATHEMATICS

(TOPICAL)

About Thinking Process

In solving mathematical problems, we always work backward. After indentifying our main target, we go 'backward' to look for the 'easier' targets until we are able to solve the problems.

Thinking process reveals how the teacher actually goes about solving a sum in the above-said manner.

About Teacher's Comments

It reveals the extra but relevant information which is not required as part of the solutions but are extremely useful in knowing how the solutions are arrived.

Period	2014 to 2025
Contents	June & November, Paper 1 & 2, Worked Solutions
Form	Topic By Topic
Compiled for	O Level
Special features	Thinking Process, Teacher's Comments

© REDSPOT PUBLISHING

Appointed Agents & Wholesalers in PAKISTAN:

BOOK LAND

Urdu Bazaar, Lahore. Tel: 042-37124656

NATIONAL BOOK SERVICE

Urdu Bazaar, Lahore. Tel: 042-37247310.

UNITED BOOKS

Urdu Bazaar Lahore. Tel: 042-37353153, 37361252

BURHANI BOOK CENTRE

New Urdu Bazaar, Karachi, Tel: 021-32634718

MARYAM ACADEMY

Urdu Bazaar, Karachi, Tel: 0331-2425264

WAQAR BROTHERS

Urdu Bazaar, Karachi, Tel: 0321-2202576

REHMAN BOOKS

College Road, Rawalpindi

Tel: 051-5770603, 0313-5618976

UNITED BOOKS

College Road, Rawalpindi,

Tel: 051-5533151, 0336-9574147

WELCOME BOOKS

Soneri Masjid Road, Peshawar Cantt.

Tel: 091-5274932, 0300-5860068

- ① Tel No: 042-35201010
- ② Mobile No: 0300-8447654
- ✉ E-Mail: info@redspot.com.pk
- 🌐 Website: redspot.pk
- 📖 E-Books: ebook.redspot.pk
- 📍 Address: P.O. Box 5041, Model Town, Lahore, Pakistan.

All rights reserved. No part of this publication may be reproduced, copied or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage and retrieval system, without permission in writing from the publisher/distributor.

Revised Syllabus

Topic 1	Numbers
Topic 1a	Everyday Mathematics
Topic 2	Exponential Growth and Decay
Topic 3	Indices, Surds and Standard Form
Topic 4	Sets and Venn Diagrams
Topic 5	Algebraic Expressions and Manipulations
Topic 6	Algebraic Fractions
Topic 7	Solutions of Equations and Simultaneous Equations
Topic 8	Inequalities
Topic 9	Sequences and Patterns
Topic 10	Direct and Inverse Proportion
Topic 11	Graphs in Practical Situations
Topic 12	Graphs of Functions
Topic 13	Functions
Topic 14	Coordinate Geometry
Topic 15	Geometrical Constructions
Topic 16	Similarity
Topic 17	Symmetry
Topic 18	Angles and Circle Properties
Topic 19	Mensuration
Topic 20	Bearings and Trigonometry
Topic 21	Transformations
Topic 22	Vectors in Two Dimensions
Topic 23	Probability
Topic 24	Statistics

Revision

 June **2025** Paper 1 & 2

December **2025** Paper 1 & 2

Topic 1a

Everyday Mathematics

1 (J2014/P1/Q14)

A rectangular garden has length 35 metres and width 25 metres.
These distances are measured correct to the nearest metre.

- Write down the upper bound of the length of the garden. [1]
- Work out the lower bound of the perimeter of the garden. [2]

Thinking Process

- Divide 1 m by 2. Add the value to the length of garden.
- To find the lower bound of the perimeter
✂ subtract 0.5 m from length and width.

Solution

- Upper bound of the length = $35 + 0.5$
= 35.5 m
- Lower bound of the perimeter
= $2(L + B)$
= $2[(35 - 0.5) + (25 - 0.5)]$
= $2(34.5 + 24.5)$
= $2(59)$
= 118 m **Ans.**

2 (J2014/P1/Q16)

- Dwayne buys a camera for \$90.
He sells the camera for \$126.
Calculate his percentage profit. [1]
- The price of a computer was \$375.
In a sale, the price was reduced by 15%.
Calculate the reduction in the price of the computer. [1]
- The exchange rate between euros and dollars is €1 = \$1.25 .
 - Convert €180 to dollars. [1]
 - Convert \$500 to euros. [1]

Thinking Process

- ✂ Find the profit earned and express it as a percentage of original cost price.
- ✂ Find 15% of \$375.

- To get €180 worth of dollars ✂ multiply €180 by 1.25.
 - Divide \$500 by 1.25

Solution

- Profit = $\$126 - \$90 = \$36$
percentage profit = $\frac{36}{90} \times 100 = 40\%$ **Ans.**
- Reduction in the price = 15% of \$375
= $\frac{15}{100} \times \$375$
= $\$ \frac{225}{4} = \56.25 **Ans.**
- €1 = \$1.25
€180 = 1.25×180
= \$225 **Ans.**
 - \$1.25 = €1
\$500 = $\left(\frac{1}{1.25} \times 500\right)$
= $\left(\frac{100}{125} \times 500\right) = €400$ **Ans.**

3 (J2014/P2/Q3)

- Mariam works in a shop.
She earns \$5.20 per hour.
She also earns a bonus of 15% of the value of the items she sells in a week.
 - In one week she works for 32 hours and sells items with a value of £2450.
Calculate Mariam's total earnings for the week. [2]
 - In another week, Mariam worked for 28 hours and earned a total of \$409.60.
Calculate the value of the items she sold that week. [3]
- Jack opens a bank account paying simple interest.
He pays in \$800 and leaves it in the account for 4 years.
At the end of 4 years he closes the account and receives \$920.
Calculate the percentage rate of simple interest paid per year. [2]
 - Jack uses some of the \$920 to pay for a holiday and a computer.
He saves the remainder.
The money is divided between the holiday, computer and savings in the ratio 4 : 5 : 7.
Calculate the amount he saves. [2]

Thinking Process

- ✂ Find the basic earning. Find the bonus. Add them together.
 - Find the basic earnings for 28 hours and subtract it from \$409.60 to find bonus. This bonus is equal to 15% of Mariam's weekly total sale. Thus, form an equation and solve it to find total sales.

- (b) (i) Find simple interest earned and then use $I = \frac{PRT}{100}$ to calculate the percentage rate.
 (ii) To find his savings, find 7 units worth of amount.

Solution

- (a) (i) Basic earning = $\$5.20 \times 32$
 $= \$166.40$
 bonus = $15\% \times \$2450 = \367.50
 $\therefore$ total earnings for the week
 $= \$166.40 + \367.50
 $= \$533.90$ **Ans.**
- (ii) Basic earning for 28 hours = $\$5.20 \times 28$
 $= \$145.60$
 $\Rightarrow$ bonus = $\$409.60 - \145.60
 $= \$264.00$
 let the value of items she sold be $\$x$
 $\therefore$ 15% of $x = \$264.00$
 $\frac{15}{100} \times x = \264.00
 $x = \$264.00 \times \frac{100}{15}$
 $= \$1760$ **Ans.**
- (b) (i) Simple Interest earned = $\$920 - \800
 $= \$120$
 $I = \frac{PRT}{100}$
 $\Rightarrow 120 = \frac{800 \times R \times 4}{100}$
 $120 = 32R$
 $R = \frac{120}{32} = 3.75\%$
 $\therefore$ rate of simple interest per year
 $= 3.75\%$ **Ans.**
- (ii) $4 + 5 + 7 = 16$
 $\therefore$ amount he saves = $\frac{7}{16} \times \$920$
 $= \$402.50$ **Ans.**

4 (N2014/P1/Q1)

Mavis went to a cafe to meet some friends.

- (a) She bought 3 drinks at \$1.42 each and 1 cake for 85 cents.
 How much did she spend altogether? [1]
- (b) She left home at 10.45 a.m. and returned at 1.20 p.m.
 How long, in hours and minutes, was she away from home? [1]

Thinking Process

- (a) Find the total cost of 3 drinks and 1 cake. $\Rightarrow$ change 85 cents into dollars.
 (b) $\Rightarrow$ Convert 1.20 p.m. to 24-hour clock reading.

Solution

with **TEACHER'S COMMENTS**

- (a) Total amount spent = $(3 \times \$1.42) + \0.85
 $= \$5.11$ **Ans.**
- (b) 1.20 p.m. = 1320
 $= 1280$
 $1280 - 1045 = 0235$
 $\therefore$ she was away from home for 2 hours and 35 minutes. **Ans.**

Note that writing 1320 as 1280 is to facilitate the subtraction.

5 (N2014/P1/Q2)

A cookery book states that the time it takes to cook some meat is

13 minutes for every 500 grams of meat + 20 minutes.

- (a) Calculate the number of minutes it takes to cook 1.5 kg of meat. [1]
- (b) It takes T minutes to cook M grams of meat. Find a formula for T . [1]

Thinking Process

- (a) 500 grams take 13 minutes to cook. Find how long would 1.5 kg take. $\Rightarrow$ Convert 1.5 kg into grams.
 (b) Form a formula according to the given statement.

Solution

- (a) 1.5 kg = $1.5 \times 1000 = 1500$ grams.
 500 grams — 13 minutes
 1500 grams — $\frac{13}{500} \times 1500 = 39$ minutes
 $\therefore$ total cooking time = $39 + 20$
 $= 59$ minutes **Ans.**
- (b) $T = \frac{13}{500}M + 20$ **Ans.**

6 (N2014/P1/Q7)

A car travels at 90 km/h.

How many metres does it travel in 1 second? [2]

Thinking Process

$\Rightarrow$ Express the speed in m/s.

Solution

$$90 \text{ km/h}$$

$$= 90 \frac{\text{km}}{\text{h}}$$

$$= 90 \times \frac{1000 \text{ m}}{3600 \text{ s}} = 25 \text{ m/s}$$

$\therefore$ the car travels 25 metres in 1 second **Ans.**

Note that,
 1 km = 1000 metres.
 1 hour = 60×60
 $= 3600$ seconds

7 (N2014/P1/Q9)

The time taken to run a race is given as 54.3 seconds, correct to the nearest 0.1 of a second.

- (a) Find the lower bound for the time taken. [1]
 (b) The distance run is given as d metres, correct to the nearest metre.

Write down an expression, in terms of d , for the maximum possible average speed, in metres per second. [1]

Thinking Process

- (a) ✎ Divide 0.1 by 2. Subtract the value from 54.3.
 (b) For greatest possible speed, increase the numerator by 0.5 m and decrease the denominator by 0.05 seconds.

Solution

- (a) Lower bound for the time taken = $54.3 - 0.05$
 $= 54.25$ s. **Ans.**
 (b) Maximum possible average speed
 $= \frac{\text{maximum distance}}{\text{minimum time}}$
 $= \frac{d + 0.5}{54.25}$ m/s **Ans.**

8 (N2014/P2/Q1)

- (a) In 2013, Mary worked for Company A. Her salary for the year was \$18750.
 (i) \$5625 of her salary was not taxed. What percentage of her salary was not taxed? [2]
 (ii) The remaining \$13125 of Mary's salary was taxed. 22% of this amount was deducted for tax. Mary's take-home pay was the amount remaining from \$18750 after tax had been deducted. She received this in 52 equal amounts as a weekly wage. Calculate Mary's weekly wage. [3]
 (iii) In 2012 Mary had worked for Company B. When she moved from Company B to Company A, her salary increased by 25% to \$18750. Calculate her salary when she worked for Company B. [2]
 (b) The rate of exchange between pounds (£) and Indian rupees (R) is £1 = R87.21. The rate of exchange between pounds (£) and Swiss francs (F) is £1 = F1.53.
 (i) Mavis changed £750 into Indian rupees. How many rupees did she receive? [1]
 (ii) David changed F450 into pounds. How many pounds did he receive? [1]
 (iii) Brian changed R50 000 into Swiss francs. How many Swiss francs did he receive? [2]

Thinking Process

- (a) (i) Express \$5625 as a percentage of \$18750 and simplify.
 (ii) Calculate the tax and subtract it from \$13125. Add the remaining amount to \$5625 to find take-home salary. Divide this amount by 52 to find weekly wage.
 (iii) ✎ 125% represent \$18750. Find 100% of the amount.
 (b) (i) ✎ Multiply £750 by 87.21
 (ii) ✎ Divide F450 by 1.53
 (iii) Note that R87.21 = F1.53

Solution

- (a) (i) $\frac{5625}{18750} \times 100 = 30\%$ **Ans.**
 (ii) Amount of tax deducted = $\frac{22}{100} \times \$13125$
 $= \$2887.5$
 remaining amount = $\$13125 - \2887.5
 $= \$10237.5$
 total take-home pay = $\$5625 + \10237.5
 $= \$15862.5$
 $\therefore$ weekly wage = $\frac{15862.5}{52} = \$305.05$
 $\approx \$305$ **Ans.**
 (iii) 125% — \$18750
 100% — $\$ \frac{18750}{125} \times 100 = \15000
 $\therefore$ salary in company B = \$15000 **Ans.**
 (b) (i) £1 — R87.21
 £750 — $R87.21 \times 750$
 $= R65407.50$ **Ans.**
 (ii) 1.53F — £1
 450F — $\£ \frac{1}{1.53} \times 450$
 $= \£294.118 \approx \£294$ **Ans.**
 (iii) From the given information, we see that,
 R87.21 — F1.53
 $\Rightarrow R50000$ — $F\left(\frac{1.53}{87.21} \times 50000\right)$
 $= F877.193 \approx F877$ **Ans.**

9 (J2015/P1/Q5)

Fariza travels from London to Astana. The time in Astana is 5 hours ahead of the time in London, so when it is 10 00 in London the local time in Astana is 15 00 . She flies from London to Moscow and then from Moscow to Astana. The flight leaves London at 12 25 and takes 4 hours to reach Moscow.

Fariza waits $4\frac{1}{2}$ hours in Moscow for the flight to Astana.
She arrives in Astana at 05 25 local time.
How long did the flight from Moscow to Astana take? [2]

Thinking Process

To find the length of flight ✎ Find the departure time from Moscow in London local time.
Find the arrival time in Astana as per London local time and then subtract the time difference.

Solution

Flight duration from London to Moscow = 4 hours
waiting time in Moscow = 4 hours 30 minutes.
∴ Time of departure from Moscow as per London
local time = $1225 + 0400 + 0430$
= 2055

When it is 0525 in Astana, the local time in London is = $0525 - 0500$
= 0025 or 2425

length of flight from Moscow to Astana,
= $2425 - 2055$
= $2385 - 2055 = 0330$

∴ the flight takes 3 hours 30 minutes from Moscow to Astana. **Ans.**

10 (J2015/P1/Q19)

- (a) Luis works in an office.
For normal time he is paid \$8 per hour.
For overtime he is paid the same rate as normal time plus an extra 50%.
One month he works 140 hours normal time and 10 hours overtime.
Work out how much he is paid for that month's work. [2]
- (b) Sara invests \$240 in an account that pays 3% per year simple interest.
She leaves the money in the account for 5 years.
Work out how much money Sara has at the end of 5 years. [2]

Thinking Process

- (a) Find the normal time payment and overtime payment separately and then add them to find the payment for the month.
- (b) Use $I = \frac{PRT}{100}$ to calculate the interest earned and then add the interest earned to the principal amount invested.

Solution

- (a) Normal time payment per hour = \$8
Overtime payment per hour = $\$8 + \frac{50}{100} \times \8
= $\$8 + \$4 = \$12$
Payment for 140 hours normal time = $\$8 \times 140$
= \$1120
Payment for 10 hours over time = $\$12 \times 10$
= \$120
Total payment made = $\$1120 + \120
= \$1240 **Ans.**
- (b) Simple interest = $\frac{PRT}{100}$
= $\frac{240 \times 3 \times 5}{100} = \36
∴ total money in Sara's account = $\$240 + \36
= \$276 **Ans.**

11 (J2015/P1/Q22)

The scale of a map is 1 : 25 000 .

- (a) The scale can be written as 1 cm : d km .
Find d . [1]
- (b) The distance between two villages is 8 km.
Find the distance, in centimetres, between the two villages on the map. [1]
- (c) The distance between the peaks of two mountains is measured on the map as 76 mm.
Calculate the distance, in kilometres, between the two peaks. [2]

Thinking Process

- (a) Express 25000 cm as km.
(b) Use the given scale to find the distance in cm.
(c) Express 76 mm in terms of cm and use the scale to find the distance.

Solution

- (a) 1 : 25000
⇒ 1 cm : 25000 cm
1 cm : $\frac{25000}{100000}$ km
1 cm : 0.25 km
∴ $d = 0.25$ km **Ans.**
- (b) 0.25 km — 1 cm
8 km — $\frac{1}{0.25} \times 8$
= $\frac{100}{25} \times 8$
= 32 cm **Ans.**

- (c) 76 mm = 7.6 cm

given scale is:

$$1 \text{ cm} \text{ — } 0.25 \text{ km}$$

$$\therefore 7.6 \text{ cm} \text{ — } 0.25 \times 7.6$$

$$= \frac{25}{100} \times \frac{76}{10}$$

$$= \frac{19}{10} = 1.9 \text{ km} \quad \text{Ans.}$$

12 (J2015/P2/Q6)

- (a) Yuvraj and Sachin travel to England.
Yuvraj exchanges 20500 rupees and receives £250.
Sachin exchanges 26650 rupees into pounds (£) at the same exchange rate.
How many pounds does Sachin receive? [2]
- (b) Dan goes to a bank to exchange some pounds (£) for euros (€).
He has £400 which he wants to exchange.
The bank only gives euros in multiples of 5 euros.
The exchange rate is £1 = €1.17.
Find the number of euros he receives and his change from £400. [3]
- (c) Kristianne buys a fridge and a freezer in a sale.
The sale offers 15% off everything and she pays a total of \$357.
Before the sale, the freezer cost \$250.
What was the cost of the fridge before the sale? [3]

Thinking Process

- (a) 20500 rupees = £250. Using ratio concepts, find what 26650 rupees are worth in pounds.
(b) Multiply £400 by €1.17.
(c) Calculate the cost of freezer in the sale. Subtract it from \$357 to find the cost of fridge in the sale. This amount is equivalent to 85% of the actual cost of fridge. Hence find 100% of the amount.

Solution

- (a) 20500 rupees = £250

$$26650 \text{ rupees} = £ \left(\frac{250}{20500} \times 26650 \right) \\ = £325 \quad \text{Ans.}$$

- (b) £1 = €1.17

$$£400 = €(1.17 \times 400) \\ = €468$$

Given that the bank only gives euros in multiples of 5 euros

$$\therefore \text{Dan receives } €465 \quad \text{Ans.}$$

$$€465 = £ \left(\frac{465}{1.17} \right) \\ = £397.44$$

$$\therefore \text{His change is: } £400 - £397.44 \\ = £2.56 \quad \text{Ans.}$$

- (c) 85% of \$250

$$= \frac{85}{100} \times \$250 \\ = \$212.50$$

$$\therefore \text{price of freezer in the sale} = \$212.50$$

$$\Rightarrow \text{price of fridge in the sale} = \$357 - \$212.50 \\ = \$144.50$$

\$144.50 is equivalent to 85% of the actual cost of fridge,

$$85\% \text{ — } \$144.50$$

$$100\% \text{ — } \$ \frac{144.50 \times 100}{85} \\ = \$170$$

$$\therefore \text{cost of fridge before the sale} = \$170 \quad \text{Ans.}$$

13 (N2015/P1/Q2)

- (a) A trader buys 7 items for \$4.10 each and 5 items for \$6.40 each.
He sells all of them for \$10 each.
Calculate his profit. [1]
- (b) Find the simple interest on \$450 for 5 years at 4% per annum. [1]

Thinking Process

- (a) To find the profit ✍ calculate the total cost of 12 items.
(b) Simple interest = $\frac{PRT}{100}$

Solution

- (a) Cost of 7 items = $7 \times \$4.10 = \28.70

$$\text{Cost of 5 items} = 5 \times \$6.40 = \$32$$

$$\text{total cost price of 12 items} = \$28.70 + \$32 \\ = \$60.70$$

$$\text{total selling price of 12 items} = 12 \times \$10 = \$120$$

$$\therefore \text{profit} = \$120 - \$60.70 \\ = \$59.30 \quad \text{Ans.}$$

- (b) Simple interest = $\frac{PRT}{100}$

$$= \frac{450 \times 4 \times 5}{100}$$

$$= \frac{9000}{100} = \$90 \quad \text{Ans.}$$

14 (N2015/P1/Q9)

At an athletics event, Dave and Ed each threw a javelin.
Dave threw 60 m, correct to the nearest 10 metres.
Ed threw 61 m, correct to the nearest metre.

- (a) Write down the lower bound for the distance thrown by Dave. [1]
- (b) Calculate the greatest possible difference between the distance thrown by Dave and the distance thrown by Ed. [1]

Thinking Process

- (a) ✎ Divide 10 by 2. Subtract the value from 60.
 (b) For greatest possible difference, add 0.5 to 61 metres and subtract 5 from 60 metres.

Solution with **TEACHER'S COMMENTS**

- (a) $10 \text{ m} \div 2 = 5 \text{ m}$
 $\therefore$ Lower bound for the distance = $60 - 5$
 $= 55 \text{ m}$ **Ans.**
- (b) Greatest possible difference in the distances
 = greatest distance thrown by Ed
 – smallest distance thrown by Dave
 $= (61 + 0.5) - (60 - 5)$
 $= 61.5 - 55$
 $= 6.5 \text{ m}$ **Ans.**

Distance for Dave is measured correct to the nearest 10 m. therefore the error is of $\pm 5 \text{ m}$.
 Similarly distance for Ed is measured correct to nearest one metre, therefore the error is of $\pm 0.5 \text{ m}$.

15 (N2015/P1/Q14)

Meeraa went on a journey from P to Q to R .
 The first part of the journey, from P to Q , took 4 hours to travel 80 km.

- (a) Find the average speed for the journey from P to Q . [1]
 (b) In the second part of the journey, from Q to R , she travelled 45 km.
 Her average speed for both parts of the whole journey from P to Q to R was 25 km/h.
 Find the time taken for the second part of the journey, from Q to R . [2]

Thinking Process

- (a) & (b) Average speed = $\frac{\text{total distance}}{\text{total time}}$

Solution

- (a) Average speed = $\frac{\text{distance}}{\text{time}}$
 $= \frac{80}{4} = 20 \text{ km/h}$ **Ans.**
- (b) Let t be the time taken from Q to R
 Average speed = $\frac{\text{total distance}}{\text{total time}}$
 $25 = \frac{80 + 45}{4 + t}$
 $25(4 + t) = 125$
 $100 + 25t = 125$
 $25t = 25$
 $t = 1 \text{ hour.}$ **Ans.**

16 (N2015/P2/Q1)

- (a) Fatima and Mohammed buy new bikes.
 (i) Fatima buys a city bike costing \$360.
 She pays 60% of the cost then pays \$15 per month for 12 months.
 (a) How much does Fatima pay altogether? [2]
 (b) Express this amount as a percentage of the original cost. [1]
 (ii) Mohammed pays \$569.80 for a mountain bike in a sale.
 The original price had been reduced by 26%.
 Calculate the original price of the mountain bike. [2]
- (b) The rate of exchange between pounds (£) and dollars is £1 = \$1.87.
 The rate of exchange between pounds (£) and euros (€) is £1 = € x .
 Rose changed \$850 and received €550.
 Calculate x . [3]

Thinking Process

- (a) (i) (a) Calculate 60% of \$360. Multiply \$15 by 2 months to find the total monthly payments. Add the two answers.
 (ii) ✎ 74% represent \$569.80. Find 100% of the amount.
 (b) Divide 1.87 by x and make an expression for €1. Divide 850 by 550 to get €1 worth of dollars. Equate to find x .

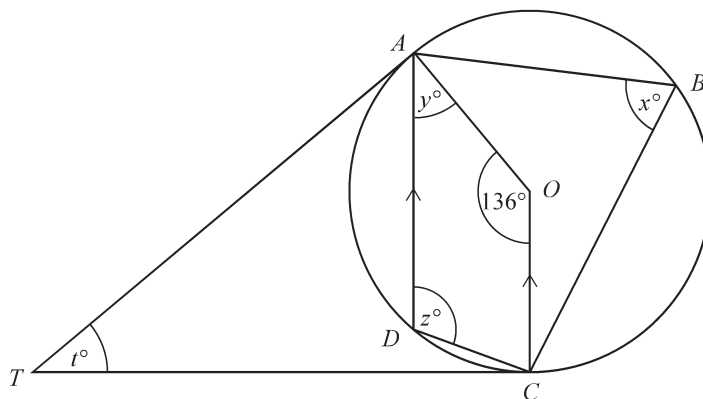
Solution

- (a) (i) (a) Total payments
 $= \text{initial payment} + \text{total monthly payments}$
 $= (60\% \times \$360) + (\$15 \times 12)$
 $= (\frac{60}{100} \times \$360) + \$180$
 $= \$216 + \180
 $= \$396$ **Ans.**
- (b) $\frac{396}{360} \times 100 = 110\%$ **Ans.**
- (ii) $100\% - 26\% = 74\%$
 $74\% \text{ — } \$569.80$
 $100\% \text{ — } \$ \frac{569.80}{74} \times 100$
 $\text{— } \$770$
 $\therefore$ original price of bike = \$770 **Ans.**
- (b) Given that, £1 = \$1.87 and £1 = € x
 $\Rightarrow$ € x = \$1.87
 $\text{€}1 = \$ \frac{1.87}{x} \dots\dots\dots(1)$
 Rose exchanged \$850 into €550
 $\Rightarrow$ €550 = \$850
 $\text{€}1 = \$ \frac{850}{550} \dots\dots\dots(2)$

Topic 18

Angles and Circle Properties

1 (N2014/P1/Q20)



In the diagram, A , B , C and D lie on the circle, centre O .

CO is parallel to DA .

The tangents to the circle at A and C meet at T .

$$\angle AOC = 136^\circ$$

- Find x . [1]
- Find y . [1]
- Find z . [1]
- Find t . [1]

Thinking Process

- $\angle AOC = 2x^\circ$. ($\angle$ at centre is twice $\angle$ at the circumference).
- $\nearrow AD$ is parallel to OC . Angles between parallel lines are supplementary.
- $\nearrow$ Note that $ABCD$ is a cyclic quadrilateral.
- $\nearrow t^\circ + \angle AOC = 180^\circ$.

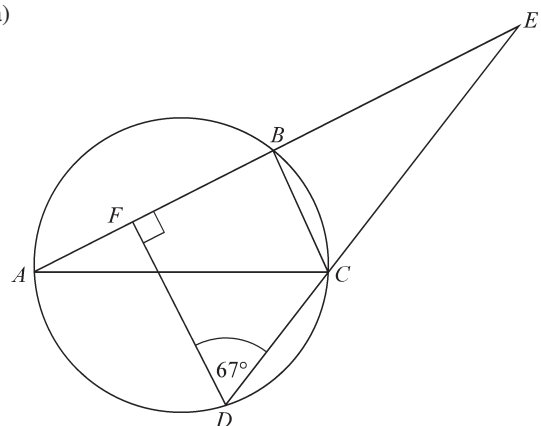
Solution

- $x^\circ = \frac{1}{2} \times 136^\circ$ ($\angle$ at centre is $2 \times \angle$ at circumference)
 $= 68^\circ$ **Ans.**
- AD is parallel to OC ,
 $\therefore y^\circ + 136^\circ = 180^\circ$ (interior $\angle$ s between $\parallel$ lines)
 $y^\circ = 180^\circ - 136^\circ$
 $= 44^\circ$ **Ans.**

- $ABCD$ is a cyclic quadrilateral,
 $x^\circ + z^\circ = 180^\circ$ (opp. $\angle$ s of a cyclic quad. are supplementary)
 $z^\circ = 180^\circ - x^\circ$
 $= 180^\circ - 68^\circ$
 $= 112^\circ$ **Ans.**
- TA and TC are tangents to the circle.
 $\therefore t^\circ + 136^\circ = 180^\circ$
 $t^\circ = 180^\circ - 136^\circ$
 $= 44^\circ$ **Ans.**

2 (N2014/P2/Q2)

(a)

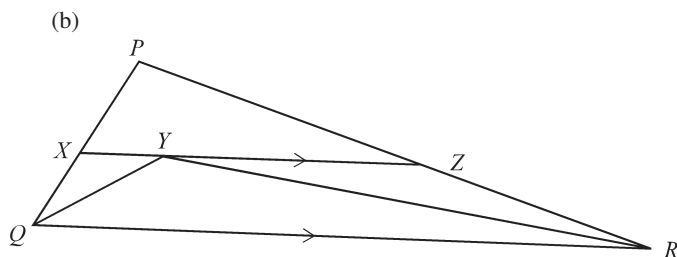


A , B , C and D are points on the circumference of the circle and AC is a diameter.

$AFBE$ and DCE are straight lines.

DF is perpendicular to AE and $\angle CDF = 67^\circ$.

- Find $\angle AED$. [1]
 - Find $\angle CBE$, giving a reason for your answer. [1]
 - Explain why DF is parallel to CB . [1]



In the triangle PQR , the bisectors of $\widehat{PQR}$ and $\widehat{PRQ}$ intersect at Y .

The straight line XYZ is parallel to QR .

Prove that the perimeter of triangle $PXZ = PQ + PR$.

[3]

Thinking Process

- (a) (i) To find $\widehat{AED}$ find $\widehat{FED}$. Note that $\triangle FED$ is a right angled triangle.
- (ii) Note that $\widehat{ABC} = 90^\circ$ and $\widehat{CBE}$ is adjacent to $\widehat{ABC}$.
- (iii) Angle $\widehat{DFB}$ is equal to angle $\widehat{CBE}$ which satisfies the property of corresponding angles.
- (b) To find the perimeter Prove that triangle XYQ and triangle ZYR are isosceles triangles.

Solution

- (a) (i) Consider right angled $\triangle FED$
- $$\widehat{FED} = 180^\circ - 90^\circ - 67^\circ = 23^\circ$$
- $\therefore \widehat{AED} = 23^\circ$ Ans.
- (ii) $\widehat{CBE} = 90^\circ$ because $\widehat{ABC} = 90^\circ$. Angle subtended at the circumference by the diameter is always 90° . $\widehat{ABC} = 90^\circ$ as it is subtended by diameter AC .
- (iii) $\widehat{DFB} = \widehat{CBE} = 90^\circ$
- since the two corresponding angles are congruent, therefore the lines DF and CB are parallel.
- (b) QY is the bisector of $\widehat{PQR}$
- $$\Rightarrow \widehat{YQR} = \widehat{XQY}$$
- given that XZ is parallel to QR
- $$\Rightarrow \widehat{YQR} = \widehat{XQY} \quad (\text{alternate angles})$$
- $$\therefore \widehat{XQY} = \widehat{XQY}$$
- $$\Rightarrow \triangle XYQ \text{ is an isosceles triangle.}$$
- $$\therefore XY = XQ \dots\dots\dots (1)$$

similarly,

RY is the bisector of $\widehat{PRQ}$

$$\Rightarrow \widehat{YRQ} = \widehat{ZRY}$$

XZ is parallel to QR

$$\Rightarrow \widehat{YRQ} = \widehat{ZRY} \quad (\text{alternate angles})$$

$$\therefore \widehat{ZRY} = \widehat{ZRY}$$

$\Rightarrow \triangle ZRY$ is an isosceles triangle.

$$\therefore YZ = ZR \dots\dots\dots (2)$$

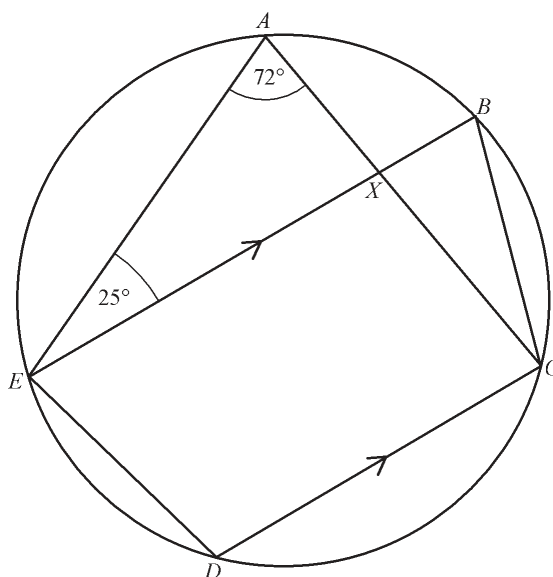
perimeter of $\triangle PXZ = PX + XY + YZ + PZ$

using results (1) and (2),

$$= PX + XQ + ZR + PZ$$

$$= PQ + PR \quad \text{Proved.}$$

3 (J2015/P2/Q4 a)



A, B, C, D and E are five points on the circumference of a circle.

EB is parallel to DC , $\widehat{EAC} = 72^\circ$ and

$$\widehat{AEB} = 25^\circ.$$

X is the intersection of AC and EB .

Find

- | | |
|-------------------------|-----|
| (i) $\widehat{EBC}$, | [1] |
| (ii) $\widehat{CXB}$, | [1] |
| (iii) $\widehat{EDC}$, | [1] |
| (iv) $\widehat{ACD}$, | [1] |

Thinking Process

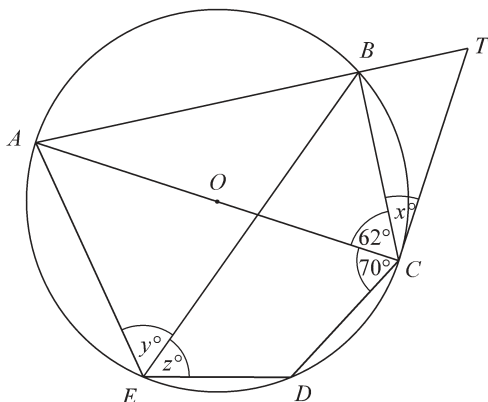
- (i) $\widehat{EBC}$ and $\widehat{EAC}$ are angles in the same segment.

- (ii) To find $\angle C\hat{X}B$ Find $\angle A\hat{X}E$. Apply sum of angles in a triangle
 (iii) $ACDE$ is a cyclic quadrilateral.
 $\angle$ opposite angles are supplementary.
 (iv) EB is parallel to DC . Apply rule of alternate angles to find angle ACD .

Solution

- (i) $\angle EBC = \angle EAC$ ($\angle$ s in the same segment)
 $= 72^\circ$
 (ii) In $\triangle AXE$,
 $\angle A\hat{X}E + 25^\circ + 72^\circ = 180^\circ$ ($\angle$ sum of a $\triangle$)
 $\angle A\hat{X}E = 180^\circ - 25^\circ - 72^\circ$
 $= 83^\circ$
 $\angle C\hat{X}B = \angle A\hat{X}E$ (vert. opp $\angle$ s)
 $= 83^\circ$ Ans.
 (iii) $ACDE$ is a cyclic quadrilateral,
 $\therefore \angle EDC + 72^\circ = 180^\circ$ (opp. $\angle$ s of cyclic quad.)
 $\angle EDC = 180^\circ - 72^\circ$
 $= 108^\circ$ Ans.
 (iv) EB is parallel to DC ,
 $\Rightarrow \angle BXC = \angle DCX = 83^\circ$ (alternate $\angle$ s)
 $\therefore \angle ACD = 83^\circ$ Ans.

4 (N2015/P1/Q17)



In the diagram, A, B, C, D and E lie on the circle, centre O .
 AC is a diameter.
 The tangent to the circle at C meets the line AB produced at T .
 $\angle ACB = 62^\circ$ and $\angle ACD = 70^\circ$.

- (a) Find x . [1]
 (b) Find y . [1]
 (c) Find z . [1]

Thinking Process

- (a) Note that $\angle ACT$ is a right angle.
 (b) $\angle AEB$ and $\angle ACB$ are angles in the same segment.
 (c) $BCDE$ is a cyclic quadrilateral $\angle$ opposite angles are supplementary.

Solution

- (a) $\angle ACT = 90^\circ$ (radius $\perp$ tangent)
 $\therefore x^\circ + 62^\circ = 90^\circ$
 $x^\circ = 90^\circ - 62^\circ$
 $= 28^\circ$ Ans.
 (b) $\angle AEB = \angle ACB$ ($\angle$ s in the same segment)
 $\therefore y^\circ = 62^\circ$ Ans.
 (c) $BCDE$ is a cyclic quadrilateral,
 $z^\circ + 70^\circ + 62^\circ = 180^\circ$ (opp. $\angle$ s of a cyclic quad. are supplementary)
 $z^\circ + 132^\circ = 180^\circ$
 $z^\circ = 180^\circ - 132^\circ$
 $= 48^\circ$ Ans.

5 (N2015/P1/Q19)

All the angles of a polygon are either 155° or 140° .
 There are twice as many angles of 155° as 140° .
 Find the number of sides of the polygon. [3]

Thinking Process

Find the exterior angles associated with 155° and 140° .
 Then use the given information to find the number of exterior angles of the polygon. Subsequently find the number of sides of the polygon

Solution with TEACHER'S COMMENTS

$$\begin{aligned} \text{Exterior angle associated with } 155^\circ &= 180^\circ - 155^\circ \\ &= 25^\circ \end{aligned}$$

$$\begin{aligned} \text{Exterior angle associated with } 140^\circ &= 180^\circ - 140^\circ \\ &= 40^\circ \end{aligned}$$

Let the number of 40° exterior angles of the polygon be x . Then, the number of 25° exterior angles will be $2x$.

$$\text{sum of exterior angles of a polygon} = 360^\circ$$

$$\Rightarrow 40x + 25(2x) = 360$$

$$40x + 50x = 360$$

$$90x = 360$$

$$x = 4$$

$$\therefore \text{number of } 40^\circ \text{ exterior angles} = 4$$

$$\text{and number of } 25^\circ \text{ exterior angles} = 2(4) = 8$$

$$\text{total number of exterior angles of the polygon}$$

$$= 4 + 8 = 12$$

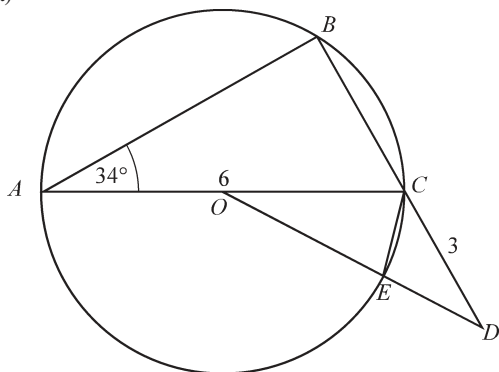
$$\therefore \text{number of sides of polygon} = 12 \text{ Ans.}$$

Note that,

- A polygon has the same number of exterior angles as it has interior angles.
- A polygon has the same number of exterior angles as it has sides.

6 (N2015/P2/Q3)

(a)



AC is a diameter of the circle, centre O.

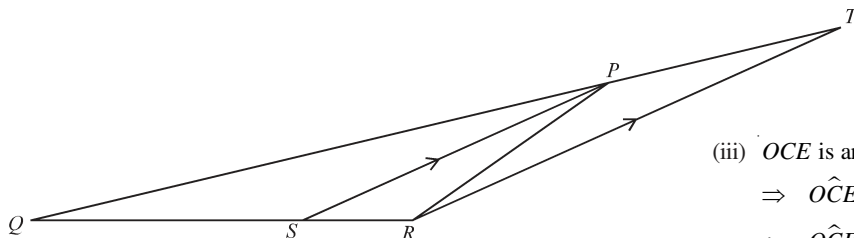
BCD and OED are straight lines.

AC = 6 cm and CD = 3 cm.

$\hat{BAC} = 34^\circ$.

- Explain why $\hat{BCA} = 56^\circ$. [1]
- Find $\hat{COD}$. [2]
- Find $\hat{OCE}$. [1]

(b)

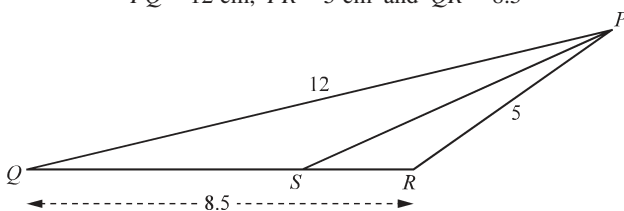


In the diagram, PS is the bisector of $\hat{QPR}$.

QPT and QSR are straight lines.

RT is parallel to SP.

- Explain why $PT = PR$. [2]
- This diagram shows part of the above diagram.
PQ = 12 cm, PR = 5 cm and QR = 8.5



It is given that $\frac{PQ}{PR} = \frac{QS}{SR}$.

Find SR.

[3]

Thinking Process

- Observe that AC is the diameter and therefore angle ABC is a right angle.
 - Observe that triangle COD is isosceles. Hence $\angle COD = \angle CDO$.
 - COE is an isosceles triangle.
- Observe that PTR is an isosceles triangle. Prove $\angle PRT = \angle PTR$.
 - Substitute the values of PQ, PR and QS to find SR.

Solution

- Given that AC is the diameter,

$$\therefore \hat{ABC} = 90^\circ$$

In $\triangle ABC$,

$$\hat{BCA} + \hat{BAC} = 90^\circ$$

$$\hat{BCA} + 34^\circ = 90^\circ$$

$$\hat{BCA} = 56^\circ$$

- $OC = CD = 3$ cm (radius of circle)

$\therefore OCD$ is an isosceles triangle.

$$\hat{COD} + \hat{CDO} = \hat{BCA} \quad \text{Ext. } \angle \text{ of a } \Delta = \text{sum of opp. int. } \angle \text{s.}$$

$$2\hat{COD} = 56 \quad (\because \hat{COD} = \hat{CDO})$$

$$\hat{COD} = \frac{56}{2} = 28^\circ \quad \text{Ans.}$$

- OCE is an isosceles triangle.

$$\Rightarrow \hat{OCE} = \hat{OEC} \quad \text{(base } \angle \text{s of isosceles } \Delta)$$

$$\therefore \hat{OCE} + \hat{OEC} + \hat{COE} = 180^\circ$$

$$2\hat{OCE} + 28^\circ = 180^\circ$$

$$\hat{OCE} = \frac{180^\circ - 28^\circ}{2} = 76^\circ \quad \text{Ans.}$$

- SP is parallel to RT

$$\Rightarrow \hat{SPR} = \hat{PTR} \quad \text{(alternate } \angle \text{s)}$$

$$\hat{QPS} = \hat{PTR} \quad \text{(corresponding } \angle \text{s)}$$

PS is the angle bisector of QPR

$$\Rightarrow \hat{QPS} = \hat{SPR}$$

$$\therefore \hat{PRT} = \hat{PTR}$$

$\Rightarrow \triangle PRT$ is isosceles.

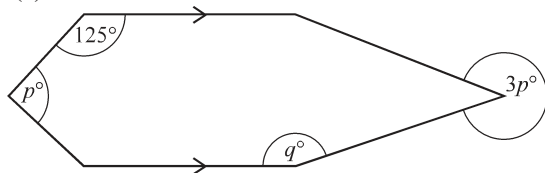
$$\therefore PT = PR$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{PQ}{PR} &= \frac{QS}{SR} \\
 \frac{12}{5} &= \frac{8.5 - SR}{SR} \\
 12SR &= 5(8.5 - SR) \\
 12SR &= 42.5 - 5SR \\
 17SR &= 42.5 \\
 SR &= 2.5 \quad \text{Ans.}
 \end{aligned}$$

7 (J2016/P2/Q4b)

- (a) Calculate the interior angle of a regular 12-sided polygon. [2]

(b)

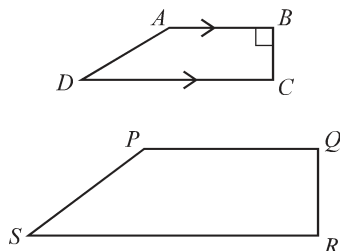


The diagram shows a hexagon with two parallel sides and one horizontal line of symmetry.

- (i) Calculate p . [1]

- (ii) Calculate q . [2]

(c)



Trapezium $PQRS$ is similar to trapezium $ABCD$.

AB is parallel to DC and $\hat{ABC} = 90^\circ$.

$$DC = 2AB, \quad BC = \frac{1}{2}AB \quad \text{and} \quad PQ = \frac{3}{4}DC.$$

Given that $BC = x$ cm, find an expression, in terms of x , for the area of $PQRS$. [3]

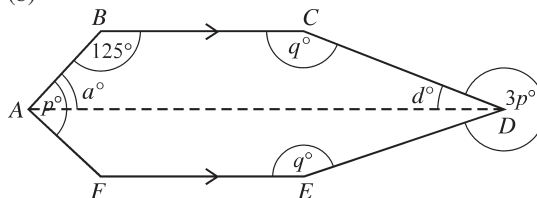
Thinking Process

- (a) ✎ Apply, one interior angle $= \frac{(n-2)180}{n}$.
- (b) (i) Draw the line of symmetry and consider the interior angles between parallel lines.
 (ii) To find q° ✎ apply sum of angles in a quadrilateral $= 360^\circ$.
- (c) Find the lengths of AB , BC and DC . Use the concept of area of similar figures to find the area of $PQRS$.

Solution

$$\begin{aligned}
 \text{(a)} \quad \text{Interior angle} &= \frac{(12-2)180^\circ}{12} \\
 &= \frac{1800^\circ}{12} = 150^\circ \quad \text{Ans.}
 \end{aligned}$$

(b)



- (i) $a^\circ + 125^\circ = 180^\circ$ (interior $\angle$ s between $\parallel$ lines)

$$\begin{aligned}
 a^\circ &= 180^\circ - 125^\circ \\
 &= 55^\circ
 \end{aligned}$$

$$\therefore p^\circ = 2 \times 55^\circ = 110^\circ \quad \text{Ans.}$$

- (ii) $\angle CDE = 360^\circ - 3p^\circ$
 $= 360^\circ - 3(110^\circ) = 30^\circ$

since AD is the line of symmetry,

$$\therefore d^\circ = \frac{30^\circ}{2} = 15^\circ$$

also, $\angle BCD = q^\circ$

now, consider quadrilateral $ABCD$,

$$a^\circ + 125^\circ + q^\circ + d^\circ = 360^\circ$$

$$55^\circ + 125^\circ + q^\circ + 15^\circ = 360^\circ$$

$$195^\circ + q^\circ = 360^\circ$$

$$q^\circ = 165^\circ \quad \text{Ans.}$$

- (c) Given that, $BC = x$ cm, we have,

$$BC = \frac{1}{2}AB \Rightarrow AB = 2BC = 2x$$

$$DC = 2AB \Rightarrow DC = 2(2x) = 4x$$

$$PQ = \frac{3}{4}DC \Rightarrow PQ = \frac{3}{4}(4x) = 3x$$

$$\text{area of } ABCD = \frac{1}{2}(x)(2x + 4x)$$

$$= \frac{1}{2}(x)(6x) = 3x^2$$

given that, $ABCD$ is similar to $PQRS$

$$\frac{\text{area of } PQRS}{\text{area of } ABCD} = \left(\frac{PQ}{AB}\right)^2$$

$$\frac{\text{area of } PQRS}{3x^2} = \left(\frac{3x}{2x}\right)^2$$

$$\text{area of } PQRS = \frac{9}{4} \times 3x^2 = \frac{27}{4}x^2 \text{ cm}^2 \quad \text{Ans.}$$